

J1. Andrew has forgotten the code to allow him to unlock his bicycle.

He knows:

- it is a 3-digit number;
- the sum of the digits is 13;
- the outer digits are even;
- subject to these conditions, the product of the digits is as large as possible.

What is the code? Explain clearly.

Solution

The middle digit is odd so it can only be 1, 3, 5, 7 or 9.

The first digit is 2, 4, 6 or 8. The cases where the first or third digit is zero give a zero product and are not relevant.

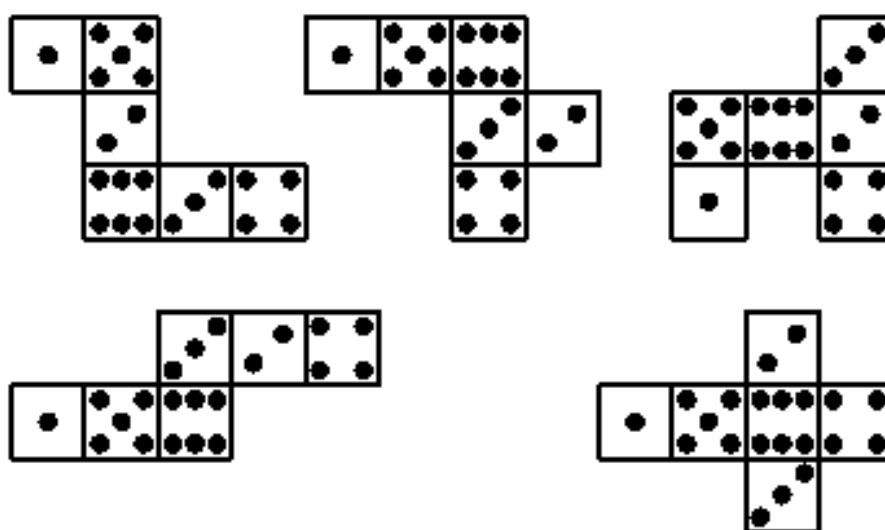
The table shows possible codes with an odd middle digit and a digit sum of 13. The product is shown in brackets

2 1 impossible	2 3 8 (48)	2 5 6 (60)	2 7 4 (56)	2 9 2 (36)
4 1 8 (32)	4 3 6 (72)	4 5 4 (80)	4 7 2 (56)	4 9 impossible
6 1 6 (36)	6 3 4 (72)	6 5 2 (60)	6 7 impossible	6 9 impossible
8 1 4 (32)	8 3 2 (48)	8 5 impossible	8 7 impossible	8 9 impossible

The biggest product is 80 which is the outcome from the triple 4 5 4.

So the code is 4 5 4.

- J2.** A cube can be opened out into a net made up of six squares. But which nets below made up of six squares fold up into a cube? For those that do not, use the spots to explain why not. For those that do, with the given spots, which form a standard dice? Explain.



Solution

Numbering the top 3 the nets 1 to 3 and the others 4 and 5, we have:

1. Not a cube as faces with 4 and 5 dots overlap.
2. Not a cube as faces with 1 and 4 dots overlap.
3. Not a cube as faces with 1 and 4 dots overlap.
4. A cube and a dice as faces with 1 and 6, 2 and 5, 3 and 4 dots are opposites.
5. A cube but not a dice as faces with 2 and 3 dots are opposites (so are 4 and 5).

The last two nets will make cubes but only the first net in the bottom row will be a dice.

- J3.** Twins Justin and Julie decided to have a snack; Justin bought a can of coke for himself and Julie bought a carton of juice and a biscuit. All three items together cost £1.60.

On their way home, Justin noted that if Julie hadn't bought the biscuit, he could have had the cake he fancied and the same amount of money would have been spent. "But," said Julie "that wouldn't have been fair since then we would have spent on you, just 20p short of five times what we would have been spent on me. As it was, exactly the same amount was spent on each of us."

What was the price of each of the items they purchased?

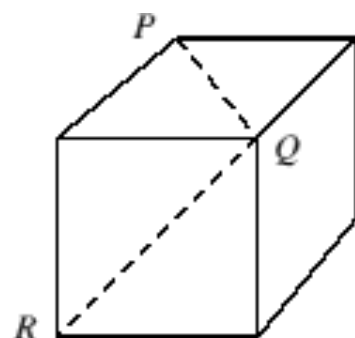
Solution

Let c , j and b be the prices in pence of each item. Then $c + j + b = 160$.

The cake and biscuit cost the same. Also, $c + b = 5j - 20$ and $c = j + b$.

Hence, $2c = 160$ so that $c = 80$ and $j = 30$, $b = 50$.

- J4.** PQ and QR are diagonals on two faces of a cube as shown. What is the size of $\angle PQR$?



Solution

All the face diagonals of a cube are equal so $PR = PQ = QR$. So PQR is an equilateral triangle, therefore $\angle PQR = 60^\circ$.

- J5.** Very shortly after leaving Elmouth by bus on the half-hour ride to Elwick one day, we met a bus coming towards us. I wondered to myself how many such buses we should meet before we reached our destination. There is a ten minute service each way. I assumed that all buses concerned travelled on time and at a constant speed. How many buses should we have met before we reached our destination?

Solution

As the buses depart at 10 minute intervals and travel towards each other, there will be 'passings' every 5 minutes.

Let the first passing be after y minutes.

It must be the case that $y < 5$ otherwise it would not be the first encounter.

So, buses will pass as follows: $y, y + 5, y + 10, y + 15, y + 20, y + 25$. These are all less than 30 but the next one would exceed 30.

So there will be 6 buses.